Online Leaving Cert maths Seminar

June 1st National College Ireland

Paper 2

Coordinate Geometry of the line

Question 2; Coordinate Geometry of the Line.

The Question can be asked in two ways .

(i)A question based on given points.

To answer this type of question you need the following formulae.

Given $a = (x_1, y_1), b = (x_2, y_2)$

(i) $|ab| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$ Distance formula

(ii) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$ Mid point of [ab] formula

(iii) $\frac{y_2 - y_1}{x_2 - x_1} = m$ Slope of [ab] formula

(iv) $y - y_1 = m(x - x_1)$ Equation of line ab

(v) $A = \frac{1}{2}|x_1 y_2 - y_1 x_2|$ Area of the Triangle oab o = (0,0)

(ii) A question based on lines

To answer this type of question you need to know the following .

(1) The slope of the line $ax + by + c = 0$ is $\frac{-a}{b}$

(vii) if (x_1, y_1) is on the line $ax + by + c = 0$ then $ax_1 + by_1 + c = 0$.

(if a point is on a line it works when you sub the point into the line).

(viii)To sketch a line (i)let x = 0 and find y ,(ii)let y = 0 and find x .

Plot the two points then draw your line using the two points.
The marking scheme works like this
(i) Any correct relevant formula written down is worth **3 marks**, **correctly filling in the formula is worth another 4 marks**, working out the information worth another 3 marks.

So (i) **always write out the formula**, (ii) fill in the formula. (iii) then tidy it up.

2005 Question 2 – Ordinary Level Paper 2 Question

2. **(a)** Find the distance between the two points (3, 4) and (15, 9).

(b) L is the line $3x - 4y + 12 = 0$.
L intersects the x-axis at a and the y-axis at b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) K is the line that passes through b and is perpendicular to L.
Show L and K on a co-ordinate diagram.

(iii) Find the equation of K.

(iv) The point $(2t, 3t)$ is on the line K. Find the value of t.

(c) The lines $2x - y + 3 = 0$ and $4x - y + k = 0$ intersect at a point.

(i) Find, in terms of k, the co-ordinates of the point of intersection of the lines.

(ii) For what value of k is the point of intersection on the y-axis?

2005 Question 2 – Ordinary Level Paper 2 Solution

(a) If $\underset{x_1, y_1}{a(3,4)}$, $\underset{x_2, y_2}{b(15,9)}$, $|ab| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

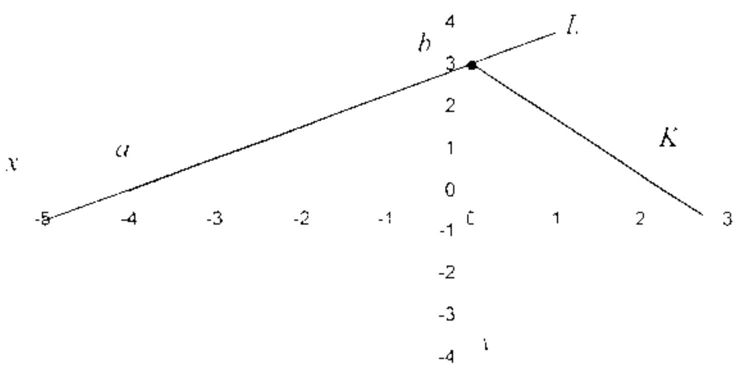
$|ab| = \sqrt{(15-3)^2 + (9-4)^2} = \sqrt{169} = 13$ **(10 marks)**

(b) $L: \; 3x - 4y - 12 = 0$

(i) To find a, let $y = 0$, $3x - 4(0) + 12 = 0 \Rightarrow 3x = -12 \Rightarrow x = -4 \; \therefore a(-4,0)$
(5 marks)

To find (b), let $x = 0$, $3(0) - 4y + 12 = 0 \Rightarrow -4y = -12 \Rightarrow y = 3 \; \therefore b(0,3)$
(5 marks)

(ii)



(10 marks)

(iii) To find the equation of K, we use $y - y_1 = m(x - x_1)$

$(x_1, y_1) = (0,3)$. The slope of L is $\dfrac{-x \text{ number}}{y \text{ number}} = \dfrac{-3}{-4} = \dfrac{3}{4}$

$\Rightarrow$ slope of K is $-\dfrac{4}{3}$ $\therefore$ equation of K is $y - 3 = -\dfrac{4}{3}(x - 0)$

$3y - 9 = -4x \Rightarrow 4x + 3y - 9 = 0$ **(5 marks)**

(iv) If $(2t, 3t)$ is on K then

$4(2t) + 3(3t) - 9 = 0 \Rightarrow 8t + 9t - 9 = 0 \Rightarrow 17t = 9 \Rightarrow t = \dfrac{9}{17}$ **(5 marks)**

(c) $2x - y = -3 \quad \times -1$ $\qquad -2x + y = 3$

$4x - y = -k$ $\qquad 4x - y = -k$

$2x = 3 - k$

$x = \dfrac{3 - k}{2}$ **(5 marks)**

Now find y using $2x - y = -3$: $2\left(\dfrac{3-k}{2}\right) - y = -3 \Rightarrow 3 - k - y = -3 \Rightarrow 6 - k = y$

$\therefore$ The point of intersection is $\left(\dfrac{3-k}{2}, 6 - k\right)$. If the lines insersect on the y axis

$\Rightarrow 3 - k = 0 \Rightarrow k = 3$ **(5 marks)**

Comment: *Why do they do this: there is no valid reason for asking (b(ii). It takes far too much time and part (c) was far too difficult for most students. Whoever set this has not spent any time recently teaching ordinary level maths*

2004 Question 2 – Ordinary Level Paper 2 Question

2. **(a)** $p(5, -8)$ and $q(11, 10)$ are two points.
Find the co-ordinates of the midpoint of $[pq]$.

(b) $a(-1, -2)$, $b(3, 1)$, $c(0, 4)$ are three points.

(i) Find the length of $[ab]$.

(ii) Calculate the area of the triangle abc.

(iii) The line L is parallel to ab and passes through the point c.
Find the equation of L.

(iv) Show that the point $d(-4, 1)$ is on L.

(v) Investigate whether $abcd$ is a parallelogram.

2004 Question 2 – Ordinary Level Paper 2 Solution

(a) $p(5,-8)$, $q(11,10)$. The midpoint formula is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+11}{2}, \frac{-8+10}{2}\right) = (8,1)$ **(10 marks)**

(b) $a(-1,-2)$, $b(3,1)$, $c(0,4)$

(i) $|ab| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(3-(-1))^2+(1-(-2))^2} = \sqrt{4^2+3^2} = \sqrt{25} = 5$

(ii) The formula for the area of a triangle is $A = \frac{1}{2}|x_1y_2 - x_2y_1|$. To use this formula one of the corners must be $(0,0)$. So move one of the corners to $(0,0)$ by a translation and then move the other corners by the same translation.

$(-1,-2)$ $(3,1)$ $(0,4)$

up 1 ↓ up 2 ↓

$(0,0)$ $(4,3)$ $(1,6)$

x_1, y_1 x_2, y_2

$\therefore A = \frac{1}{2}|4(6)-3(1)| = 10.5$ square units

(iii) Slope of L = slope of ab $= \frac{y_2-y_1}{x_2-x_1} = \frac{1--2}{3--1} = \frac{3}{4}$. Formula for equation of line is

$y - y_1 = m(x - x_1)$, $m = \frac{3}{4}$, $(x_1, y_1) = (0,4)$

$y - 4 = \frac{3}{4}(x-0) \Rightarrow y - 4 = \frac{3}{4}(x-0) \Rightarrow 4y - 16 = 3x - 0$

$\Rightarrow 3x - 4y + 16 = 0$ is the equation of L.

(iv) To show $d(-4,1)$ is on L substitute $(-4,1)$ into L

$-3(-4)+4(1)-16 = 0 \Rightarrow (-4,1)$ is on L

(v) To investigate if $abcd$ is a parallelogram, we know $ab \parallel dc$

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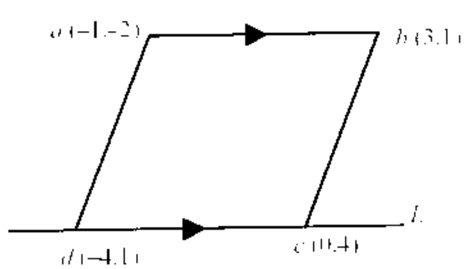


We must now show $ad \parallel bc$

Slope of $ad = \frac{1--2}{-4--1} = \frac{3}{-3} = -1$

Slope of $bc = \frac{4-1}{0-3} = -1 \Rightarrow ad \parallel bc$

and $ab \parallel dc \Rightarrow abcd$ is a parallelogram.

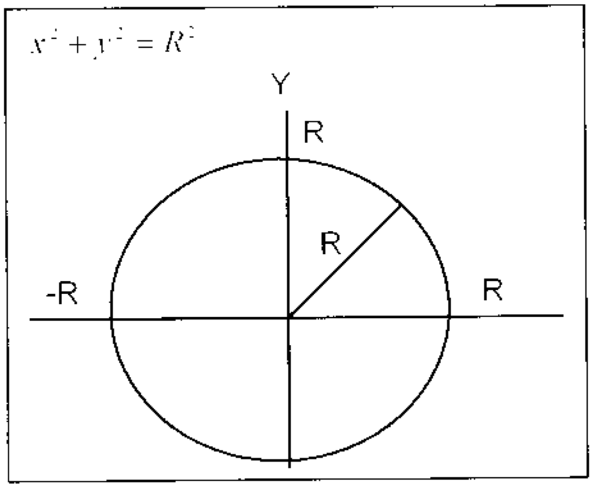
Note a parallelogram is a four-sided figure whose opposite sides are parallel.

Comment: *Nice question, no real difficulty.*

Question 3 Coordinate Geometry of The Circle

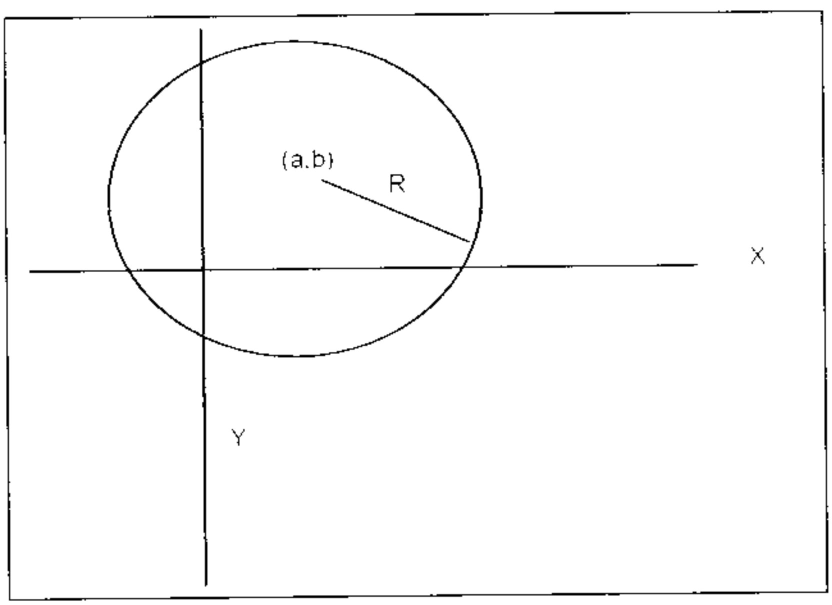
Formulae required for the question

(i) $x^2 + y^2 = R^2$ The equation of a circle centre (0,0) radius R.



Here they will ask
(i)Find the Centre and radius
(ii)show that a given point is inside/outside or on the circle
(iii)Find where the circle cuts the axes
(iv)Find where a line cuts the circle
(v)Find the equation of a tangent at a point on the circle.

(ii) $(x-a)^2 + (y-b)^2 = R^2$ The equation of a circle centre (a, b) radius R



What can be asked on this circle is limited by the Syllabus
(i)Find the centre and radius of a given circle
(ii)Given the centre and radius find the equation of the circle
(iii)given the ends of a diameter to find the equation of the circle
(iv)Given a point in the form (a,k) which is on the circle to find k .
(v)Questions based on finding the equations of tangents have been asked,(not strictly on syllabus) usually involved use of a diagram

(iii) $xx_1 + yy_1 = R^2$ The equation of the tangent at (x_1, y_1) on the circle $x^2 + y^2 = R^2$ not usually shown to Ordinary level students but ca be useful!

Again writing out each formula is worth 3 marks, filling it in 4 marks. tidy up 3 marks.

2005 Question 3 – Ordinary Level Paper 2 Question

3. **(a)** The circle C has equation $x^2 + y^2 = 49$.

(i) Write down the centre and the radius of C.

(ii) Verify that the point $(5, -5)$ lies outside the circle C.

(b) The line $y = 10 - 2x$ intersects the circle $x^2 + y^2 = 40$ at the points a and b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) Show the line, the circle and the points of intersection on a co-ordinate diagram.

(c) The circle K has equation $(x+4)^2 + (y-3)^2 = 36$.

(i) Write down the co-ordinates of the centre of K.

(ii) The point (2, 3) is one end-point of a diameter of K.
Find the co-ordinates of the other end-point.

(iii) The point $(-4, y)$ is on the circle K. Find the two values of y.

2005 Question 3 – Ordinary Level Paper 2 Solution

(a) (i) C is $x^2+y^2=49$, centre $(0,0)$ radius $=7$

$x^2+y^2=r^2$ is a circle centre $(0,0)$, radius r **(5 marks)**

(ii) To show $(5,-5)$ is outside C substitute $(5,-5)$ into $x^2+y^2=49$

$5^2+(-5)^2=50>49 \Rightarrow (5,-5)$ is outside **(5 marks)**

(b) (i) $y=10-2x$. Substitute this into $x^2+y^2=40$

$x^2+(10-2x)^2=40$

$x^2+(10-2x)(10-2x)=40$

$x^2+100-20x-20x+4x^2=40$

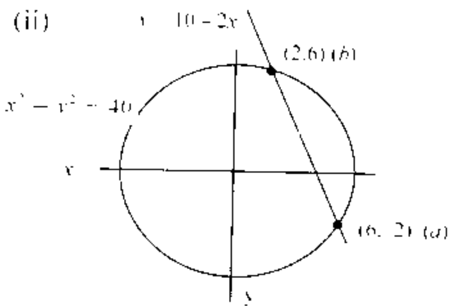
$5x^2-40x+100=40 \Rightarrow 5x^2-40x+60=0$ (divide everything by 5)

$x^2-8x+12=0 \Rightarrow (x-6)(x-2)=0 \Rightarrow x=6,\ x=2$

$y=10-2x$, when $x=6$, $y=10-2(6)=-2$ $(6,-2)(a)$

$y=10-2x$, when $x=2$, $y=10-2(2)=6$ $(2,6)(b)$

(ii)



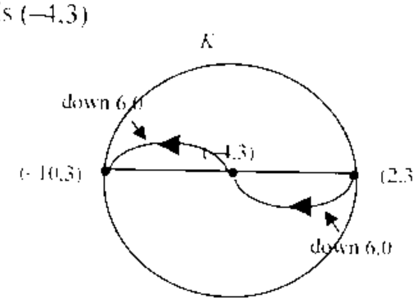
(20 marks)

(c) (i) K: $(x+4)^2+(y-3)^2=36$

This is a circle of the form $(x-a)^2+(y-b)^2=r^2$

centre (a,b), radius r. $\therefore$ Centre of K is $(-4,3)$ **(10 marks)**

(ii) To find the other end of the diameter, reflect $(2,3)$ in $(-4,3)$



(5 marks)

(iii) If $(-4,y)$ is on $(x+4)^2+(y-3)^2=36$, then replace x by -4 and y by y

$(-4+4)^2+(y-3)^2=36 \Rightarrow (y-3)(y-3)=36$

$y^2-6y+9=36 \Rightarrow y^2-6y-27=0$

$(y-9)(y+3)=0 \Rightarrow y=-3,\ y=9$ **(5 marks)**

Comment: *Apart from the fact that they asked for another diagram, some students had a problem with this as they tried to draw the circle first but since the radius is $\sqrt{40}$, they were put off. The rest of the question was fine.*

2004 Question 3 – Ordinary Level Paper 2 Question

3. **(a)** The circle C has equation $x^2 + y^2 = 36$.

(i) Write down the radius of C.

(ii) The radius of another circle is twice the radius of C.
The centre of this circle is $(0, 0)$. Write down its equation.

(b) A circle has equation $x^2 + y^2 = 13$.
The points $a(2, -3)$, $b(-2, 3)$ and $c(3, 2)$ are on the circle.

(i) Verify that $[ab]$ is a diameter of the circle.

(ii) Verify that $\angle acb$ is a right angle.

(c) K is a circle with centre $(-2, 1)$. It passes through the point $(-3, 4)$.

(i) Find the equation of K.

(ii) The point $(t, 2t)$ is on the circle K.
Find the two possible values of t.

2004 Question 3 – Ordinary Level Paper 2 Solution

(a) (i) $x^2+y^2=36$ radius = 6 (a circle centre (0,0), radius r, is $x^2+y^2=r^2$) **(5 marks)**

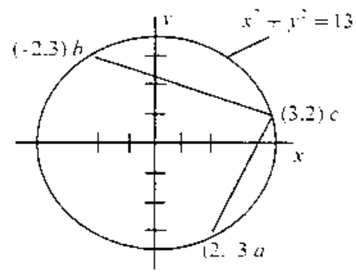
(ii) Centre (0,0), radius 12, circle is $x^2+y^2=12^2$ **(5 marks)**

(b) (i) Midpoint of $|ab| = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} = \frac{-2+2}{2}, \frac{3+-3}{2} = (0,0)$

= the centre of $x^2+y^2=13$ **(10 marks)**

(ii) To show $\angle abc = 90$ use slopes

If $L_1 \perp L_2$ then $m_1 \times m_2 = -1$



Slope of $ac = \frac{y_2-y_1}{x_2-x_1} = \frac{2--3}{3-2} = \frac{5}{1} = 5$

Slope of $bc = \frac{y_2-y_1}{x_2-x_1} = \frac{2-3}{8--2} = \frac{-1}{5}$

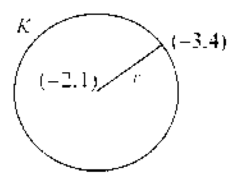
$5 \times \frac{-1}{5} = -1 \Rightarrow ac \perp bc$ **(10 marks)**

(c) (i) $r=\sqrt{(-3--2)^2+(4-1)^2}$

$r=\sqrt{-1^2+3^2}=\sqrt{10}$

(use $(x-a)^2+(y-b)^2=r^2$)

$\therefore\ K=(x+2)^2+(y-1)^2=10$ **(10 marks)**



(ii) If $(t,2t)$ is on K, then

$(t+2)^2+(2t-1)^2=10$

$(t+2)(t+2)+(2t-1)(2t-1)=10$

$t^2+4t+4+4t^2-4t+1=10$

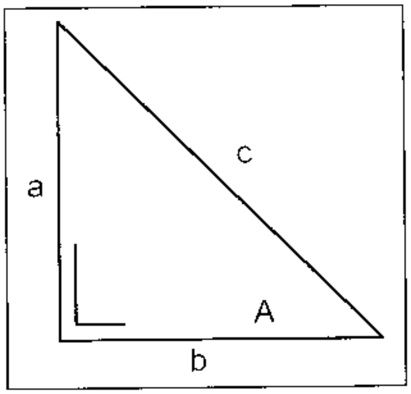
$5t^2-5=0 \Rightarrow 5(t^2-1)=0 \Rightarrow 5(t-1)(t+1)=0$

$\Rightarrow t=1,\ t=-1$ **(10 marks)**

Comment: *Lovely question, cannot really see the point of part (b).*

Question 5 Trigonometry.

Formulae needed for part (a) are.



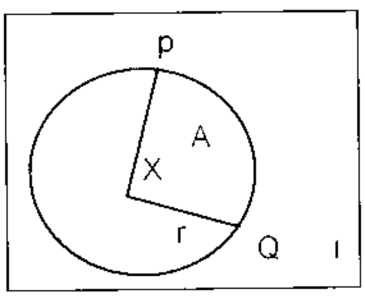
$$SinA = \frac{a}{c}, CosA = \frac{b}{c}, TanA = \frac{a}{b}$$

$$a^2 + b^2 = c^2$$

Use of the above rules is more or less confined to part (a) if the angle changes **from A to 2A** you will need to see the connection between A and 2A on **page 9** of the maths tables

None of the above are in the tables and must be learned off by heart .

Formulae needed for part b.



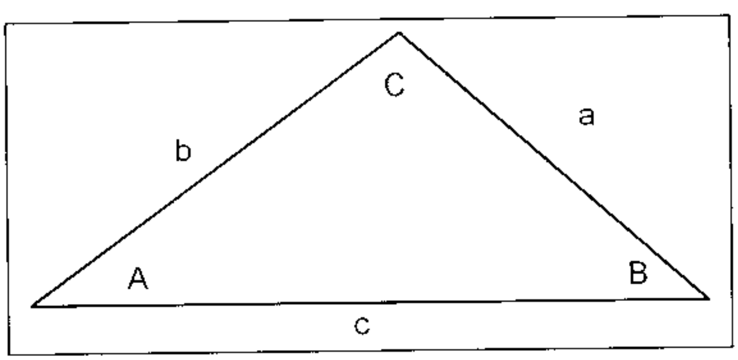
$$|PQ| = \frac{X}{360} 2\pi r$$

Length of arc PQ

$$A = \frac{X}{360} \pi r^2$$

Area of sector A.

The formula for the length of an arc and the area of a sector are on Page 8 of the tables but are in a slightly different format



Questions involving the sine and cosine rules in general appear in part c

The Sine Rule is used when you know lots of angles.

The Cosine Rules is used when you know lots of sides

In both cases always let a equal what you are looking for.

Area of a triangle $\frac{1}{2} abSinC$. **Sine Rule** $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

Cosine Rule $a^2 = b^2 + c^2 - 2bcCosA$.

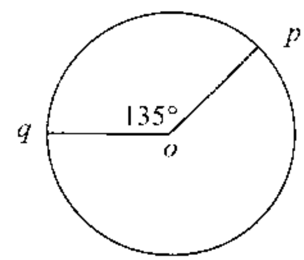
The marking scheme works like this any of the above formula filled in correctly is worth 7 marks The same rules as for coordinate geometry

2005 Question 5 – Ordinary Level Paper 2 Question

5. **(a)** A circle has centre o and radius 14 cm.

p and q are two points on the circle and $|\angle qop| = 135°$.

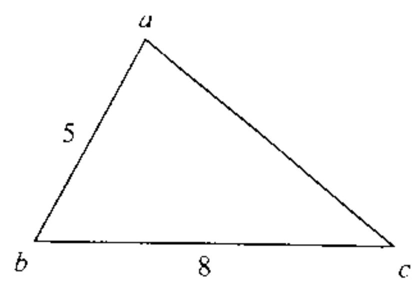
Find the length of the shorter arc pq.
Take $\pi = \frac{22}{7}$.



(b) In the triangle abc, $|ab| = 5$ cm and $|bc| = 8$ cm.
The area of the triangle is $16.58\ \text{cm}^2$.

(i) Find $|\angle abc|$, correct to the nearest degree.

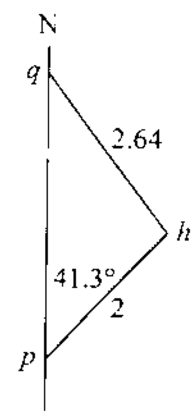
(ii) Find $|ac|$, correct to the nearest centimetre.



(c) A lighthouse, h, is observed from a ship sailing a straight course due North.
The distance from p to h is 2 km and the bearing of the lighthouse from p is N 41.3° E.
The distance from q to h is 2.64 km.

(i) Find the bearing of the lighthouse from q.

(ii) The ship is sailing at a speed of 19 km/h.
Find, correct to the nearest minute, the time taken to sail from p to q.



Marking scheme changes for 2005

Question 5(a): The marks can only be allocated as follows: (i) **10 marks**, correct answer; (ii) 7 **marks**, one error in formula or substitution (old scheme 9 marks); (iii) **3 marks**, correct answer no work or has a relevant step.

Comment: *In many situations a correct answer with no work will get full marks; see the following questions all on 2005 Paper 2 1(a), 3(a),(c), 4(a), 6(a),(b),(c), 7(a), etc.; all of the above got full marks where no work is shown. Why is trigonometry being penalised? The trigonometry question is not popular: marking it like this will not increase its popularity.*

Question 5(b)(i) The marks can only be allocated as follows: (i) **10 marks**, correct answer work shown; (ii) **9 marks**, round off omitted or incorrect; (iii) **7 marks**, the sine of the angle is found but not the angle or one error in finding the angle; (iii) **3 marks**, attempt mark or correct answer no work; (iv) **0 marks**, no step in the right direction. 5(b)(ii) The marks can only be allocated as follows: (i) **10 marks**, correct answer work shown; (ii) **9 marks**, round off error or incorrect; (iii) **7 marks**, fills in the cosine formula but makes a mess of the calculations or gets a correct answer having made mistakes in the calculations (i.e. wrong way right answer); (iv) **3 marks**, no work shown! The attempt mark.

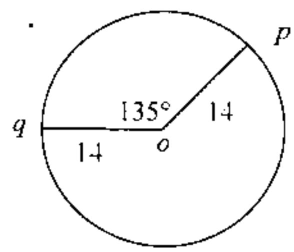
Comment: *Keeps punishing the good student and the student who makes a minor mistake. This marking scheme awards 7 marks for filling in a formula correctly but takes away 3 marks for a small error.*

Question 5(c)(ii) The marks here are allocated as above but in (c)(i) the marks are allocated as follows: (i) **10 marks**, correct answer with work shown; (ii) **7 marks**, correct formula filled in correctly or incorrectly; (iii) **3 marks**, correct answer no work or any attempt.

Comment: *You can see that they are giving marks for basically turning up in this part of the question – reason being that very few students were able to do it. In most parts of the question it really was not possible to get the right answer with no work shown, but many ex Higher course students could have arrived at most of the answers showing very little work so what amount of work must be shown?*

2005 Question 5 – Ordinary Level Paper 2 Solution

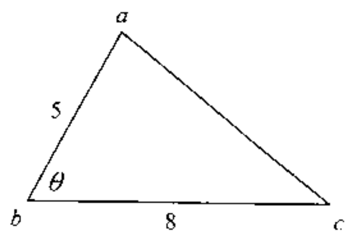
(a)



$|\text{short arc } pq| = \frac{135}{360} \cdot 2\pi r$

$\frac{135}{360} \cdot 2 \cdot \frac{22}{7} \cdot 14 = 33$ **(10 marks)**

(b)



(i) Area of a triangle is $A = \frac{1}{2} ab \sin c$

$\Rightarrow \frac{1}{2}(5)(8)\sin\theta = 16.58$

$\sin\theta = \frac{16.58}{20}$

$\sin\theta = 0.829 \Rightarrow \theta = \text{2nd fn } \sin.829 = 56°$ **(10 marks)**

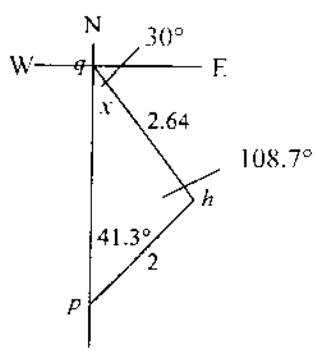
(ii) To find $|ac|$ use the cosine rule (as we know, lots of sides; always let a equal what you are looking for)

$a^2 = b^2 + c^2 - 2bc\cos A$

$|ac|^2 = 5^2 + 8^2 - 2(5)(8)\cos 56 \Rightarrow |ac|^2 = 44.26 \Rightarrow |ac| = \sqrt{44.26} = 6.65$

$|ac| = 7$ cm **(10 marks)**

(c)



(i) Use the sine rule to find x

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin x}{2} = \frac{\sin 41.3}{2.64}$

$\Rightarrow \sin x = \frac{2\sin 41.3}{2.64}$

$\Rightarrow \sin x = 0.5 \Rightarrow x = 30$

$\therefore$ The bearing of h from q is S30°E or E60°S **(10 marks)**

(ii) Complete the triangle $\angle h + 41.3 + 30 = 180° \Rightarrow h = 180 - 71.3 = 108.7$

Use the sine rule to find $|pq|$

$\frac{\sin 108.7}{|pq|} = \frac{\sin 30}{2} \Rightarrow \frac{2\sin 108.7}{\sin 30} = |pq| = 3.79$ km

$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{3.79}{19} = 0.199 = 0.2$ hours $= 12$ minutes

$\therefore$ Time taken in sailing from p to q is 12 minutes. **(10 marks)**

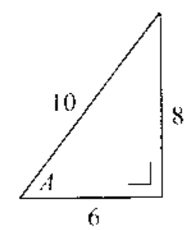
Comment: *Nice question apart from c(i). Nobody had any idea what a bearing was!*

2004 Question 5 – Ordinary Level Paper 2 Question

5. **(a)** The lengths of the sides of a right-angled triangle are shown in the diagram and A is the angle indicated.

(i) Write down the value of $\cos A$.

(ii) Hence, find the angle A, correct to the nearest degree.

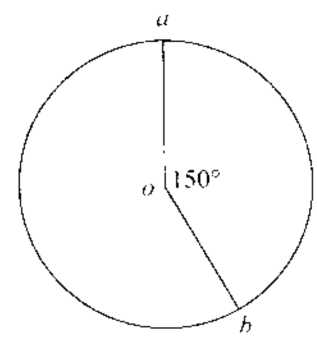


(b) A circle has centre o and radius 4 cm. a and b are two points on the circle and $|\angle aob| = 150°$.

(i) Find the area of the circle, correct to the nearest cm^2.

(ii) Find the area of the sector aob, correct to the nearest cm^2.

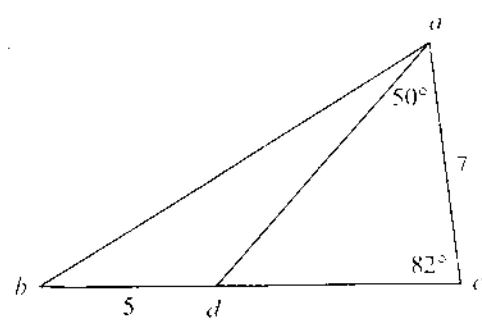
(iii) Find the length of the shorter arc ab, correct to the nearest cm.



(c) In the triangle abc, d is a point on $[bc]$. $|bd| = 5$ cm, $|ac| = 7$ cm. $|\angle dca| = 82°$ and $|\angle cad| = 50°$.

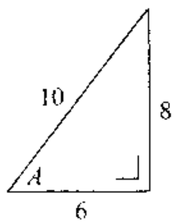
(i) Find $|dc|$, correct to the nearest cm.

(ii) Find $|ab|$, correct to the nearest cm.



2004 Question 5 – Ordinary Level Paper 2 Solution

(a)

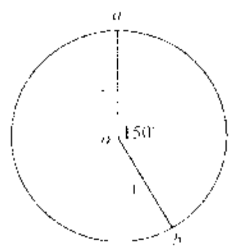


(i) $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{10}$

(ii) $A = \cos^{-1}\frac{6}{10} = 53$

(10 marks)

(b)



(i) Area of the curve is πr^2

$\pi 4^2 = 16\pi$ (??to nearest cm??)

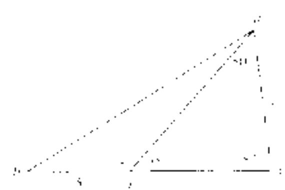
(ii) Area of the sector is

$\frac{150}{360} \times 16\pi = \frac{20}{3}\pi$ (??to nearest cm^2??) **(10 marks)**

(iii) Length of arc is $\frac{150}{360} 2\pi r$

$= \frac{150}{360} 2\pi(4) = \frac{10}{3}\pi$ (??to nearest cm??) **(10 marks)**

(c)



(i) Use the sine rule to find $|bc|$

$\frac{\sin 50}{x} = \frac{\sin 48}{7} \Rightarrow \frac{7\sin 50}{\sin 48} = x = 7.21$

$|bc| = 7$ cm **(10 marks)**

(ii) Use the cosine rule to find $|ab|$ Hint: Use the triangle abc

$ab^2 = ac^2 + bc^2 - 2|ac||bc|\cos 82°$

$ab^2 = 7^2 + 12^2 - 2(7)(12)\cos 82 = 169.62$

$\Rightarrow \quad ab = \sqrt{169.62} = 13$ cm **(10 marks)**

Comment: *Nice question if you know your stuff. Trigonometry should be on everybody's list.*