National College of Ireland

School of Computing

First Year Entrance Sample Assessment

Mathematics Sample Assessment For Different Sections

SOLUTIONS

19th, 20th and 21th August 2019

DAY 1, DAY 2 and DAY 3

All questions carry equal marks

DAY 1 (9:00 am to 5:00 pm)

Section: Probability and Statistics

Question 1:

(a) Two cards are drawn from a deck of 52 cards in such a way that the card is replaced after the first draw. Find the probabilities in the following cases:

(i). First card is King and the second is Queen
(ii). Both cards are faced cards, i.e., Kind, Queen and Jack.

(15 marks)

Solution (a):

Since there are 52 cards in the deck, so n(S) = 52

Suppose A be the event that the first card is King, then n(A) = 4
and Suppose B be the event that the second card is Queen, then n(B) = 4

(i). Now probability P(A and B) = P(A) * P(B) = n (A) / n(S) * n(B) / n(S) = 4/52 * 4/52 = 1/169
(ii). Suppose C be the event that first card is faced card.

Since there are 12 faced cards in the deck, therefore n(C) = 12
and suppose D be the event that the second card is also faced card, then n(D) = 12

probability P(C and D) = P(C) * P(D) = n (C) / n(S) * n(D) / n(S) = 12/52 * 12/52 = 9/169

(b) Three coins are tossed, each toss resulting in a head (H) or a tail (T). Make out a sample space for the possible results and write down the probability that the coins show

(i). HHH
(ii). HTH in that order
(iii). 2 heads and 1 tail in any order

(10 marks)

Solution (b):

Sample space (S) = {HHH, HTT, HHT, THT, HTH, TTH, THH, TTT}

(i). P(HHH) HHH = 1/8
(ii). P(HTH) HTH = 1/8
(iii). P(2H and 1T) = 3/8

Question 2:

Over the course of a rugby competition, a record is kept of the number of penalties conceded per game. The results are presented in the following frequency distribution:

Number of Penalties Conceded	Number of Games
0	0
1	3
2	5
3	8
4	2
5	2

Calculate the **standard deviation** of the distribution.

Solution 2:

Number of Penalties Conceded (x)	Number of Games (fx)	x * f(x)	u	x - u	$(x - u)^2$	fx * $(x - u)^2$
0	0	0	2.75	-2.75	7.5625	0
1	3	3	2.75	-1.75	3.0625	9.1875
2	5	10	2.75	-0.75	0.5625	2.8125
3	8	24	2.75	0.25	0.0625	0.5
4	2	8	2.75	1.25	1.5625	3.125
5	2	10	2.75	2.25	5.0625	10.125
	20	**55**				**25.75**
VARIANCE = 25.75/20 = 1.28						
Standard Deviation = 1.13						

Calculator can be used for the calculation.

DAY 2 (9:00 am to 5:00 pm)

Section 2: Geometry and Trigonometry

Question 3:

(a) Find two values of $\tan(x)$ for which $\cos(x) = {}^{1}/_{\sqrt{2}}$, where $0° < x \leq 360°$.

(10 marks)

Solution (a):

$$\cos(x) = {}^{1}/_{\sqrt{2}} => x = cos^{-1}\left({}^{1}/_{\sqrt{2}}\right) = 45°$$

And in the 4th quadrant is

$$x = 360° - 45° = 315°$$

Therefore, the values of tan(x) are

$$\tan(45°) = 1$$
$$\tan(315°) = -1$$

(b) The length of the perpendicular to a line from the origin is 5 units. The line passes through the point (3, 5). Find the equations of two such lines.

(15 marks)

Solution (b):

$$\text{Point}(3, 5), \qquad \text{Slope} = m$$
$$y - 5 = m(x - 3)$$
$$mx - y - 3m + 5 = 0$$

From Origin (0, 0), |PD| is calculated as

$$|PD| = \frac{|m(0) - 1(0) - 3m + 5|}{\sqrt{m^2 + (-1)^2}} = 5$$
$$|-3m + 5| = 5\sqrt{m^2 + 1}$$
$$9m^2 - 30m + 25 = 25(m^2 + 1)$$
$$16m^2 + 30m = 0$$
$$2m(8m + 15) = 0 => m(8m + 15) = 0$$
$$m = 0, \qquad m = -15/8$$

Substitute the values of m into above equation ($mx - y - 3m + 5 = 0$)

$$0x - y - 3 * 0 + 5 = 0 \text{ and } -\frac{15}{8}x - y - 3\left(-\frac{15}{8}\right) + 5 = 0$$

Therefore,

$$y - 5 = 0 \text{ and } 15x + 8y - 85 = 0$$

Section 4: Number Systems

Question 4:

(a) Show that $(-2 + 2i)$ is a root of the equation $z^3 + 3z^2 + 4z - 8 = 0$. Write the other roots.

(15 marks)

Solution (a):

$-2+2i$ is a root of $z^3+3z^2+4z-8=0$

$\Rightarrow\ (-2+2i)^3+3(-2+2i)^2+4(-2+2i)-8=0$

$(-2)^3+3(-2)^2(2i)+3(-2)(2i)^2+(2i)^3+3[+4+2(-2)(2i)+(2i)^2]-8+8i-8=0$

$-8+\cancel{24i}+24-\cancel{8i}+12-\cancel{24i}-12-8+\cancel{8i}-8=0$

$+36-36=0$

Since the coefficients of $f(z)$ are real $\Rightarrow$ the conjugate $-2-2i$ is also a root.

$z_1=-2+2i$

$z_2=-2-2i$

$\Rightarrow$ sum of roots $=(-2+2i)+(-2-2i)=-4$

also, the product of the roots $=(-2+2i)(-2-2i)=4+\cancel{4i}-\cancel{4i}-4i^2$

$=8$

$\therefore$ the quadratic formed from two roots $=z^2+4z+8=0$

$$\therefore\quad \begin{array}{r|l} & z-1 \\ \hline z^2+4z+8 & z^3+3z^2+4z-8 \\ & \underline{z^3+4z^2+8z} \\ & \quad -z^2-4z-8 \\ & \quad \underline{-z^2-4z-8} \end{array}$$

$\therefore\quad z-1$ is a factor $\Rightarrow\ z=1$ is the third root

(b) Solve the equation $z^2-2z+2=0$ and express your answer in form $r(\cos\theta+i\ \sin\theta)$.

(10 marks)

Solution (b):

$$z^2 - 2z + 2 = 0$$

$$a = 1,\ b = -2,\ c = 2$$

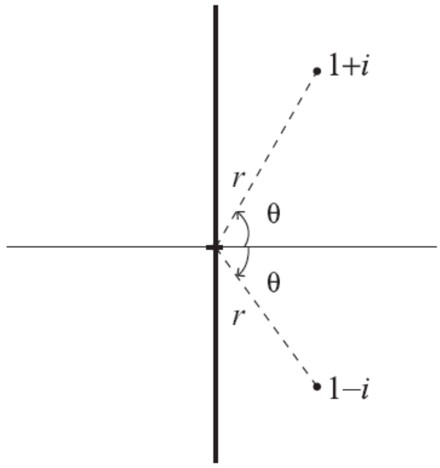
$$z = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$$

$$= 1 \pm i$$

$$r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ \left(\frac{\pi}{4}\right) \quad \text{or} \quad \left(-\frac{\pi}{4}\right)$$



$$z = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

DAY 3 (9:00 am to 5:00 pm)

Section 5: Algebra

Question 5:

(a) Find the point of intersection of each of the following sets of planes.

$$2a + b + c = 8$$
$$5a - 3b + 2c = -3$$
$$7a - 3b + 3c = 1$$

(15 marks)

Solution (a):

$$\begin{aligned}
&A:\ 2a+b+c=8 && \Rightarrow && 3A:\ 6a+\cancel{3b}+3c=24\\
&B:\ 5a-3b+2c=-3 && && B:\ \underline{5a-\cancel{3b}+2c=-3}\\
&C:\ 7a-3b+3c=1 && && D:\text{adding}:\ 11a \quad +5c=21\\
& && && \text{also}\quad B:\ 5a-\cancel{3b}+2c=-3\\
& && && C:\ \underline{7a-\cancel{3b}+3c=1}\\
& && && E:\ \text{subtracting}:\ -2a \quad -c=-4.
\end{aligned}$$

$$\begin{aligned}
&\text{since}\ \ D:\ \ 11a+\cancel{5c}=21\\
&\text{and}\ \ 5E:\underline{-10a-\cancel{5c}=-20}\\
&\text{adding}\ \ :\ \ a \quad =1\\
&\text{since}\ \ D:11a+5c=21\\
&\Rightarrow \quad 11(1)+5c=21\\
&\qquad\qquad 5c=10\\
&\qquad\qquad c=2.\\
&\text{also, } A:2a+b+c=8\\
&\qquad 2(1)+b+2=8\\
&\qquad\qquad b=4\\
&\therefore\ \ \text{solution } (a,b,c)=(1,4,2)
\end{aligned}$$

(b) Find the values of k if the equation $k^2x^2 + 2(k+1)x + 4 = 0$.

(10 marks)

Solution (b):

$$\text{Equal Roots} \Rightarrow (2k+2)^2 - 4(k^2)(4) = 0$$
$$\Rightarrow 4k^2 + 8k + 4 - 16k^2 = 0$$
$$\Rightarrow 12k^2 - 8k - 4 = 0$$
$$\Rightarrow 3k^2 - 2k - 1 = 0$$
$$\Rightarrow (3k+1)(k-1) = 0$$
$$\Rightarrow k = -\frac{1}{3}, k = 1$$

Section 6: Functions

Question 6:

(a) $Let\ f: N \to N$ with $x \mapsto 2x$ define a function.

(i). What is the domain of f?
(ii). What is the range of f?
(iii). Using the codomain and range, explain why f is not a surjective function.
(iv). Is f a one-to-one function?
(v). Suggest a restriction on the codomain to make f a surjective function.

(15 marks)

Solution (a):

(i). N (Natural Numbers) = {1, 2, 3, 4, …… }
(ii). Since *f*(x) = 2x, the range is the set of all even numbers
(iii). Codomain and Range are not equal
(iv). Since each value of x in *f*(x) corresponds to a unique value, therefore f is a one-to-one function.
(v). Codomain should be the set of even positive numbers.

(b) Find the equation of the tangent to the curve $y = lnx + x - 2$ at the point where $x = 1$.

(10 marks)

Solution (b):

$y = \ell n\, x + x - 2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x} + 1$

where $x = 1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1} + 1 = 2$ (slope)

where $x = 1 \Rightarrow y = \ell n(1) + 1 - 2 = 0 - 1 = -1$

$\Rightarrow$ Point $(1, -1)$

$\Rightarrow$ Equation of Tangent : $y + 1 = 2(x - 1)$

$\Rightarrow \quad y + 1 = 2x - 2$

$\Rightarrow \; 2x - y - 3 = 0$