

National College of Ireland

School of Computing
First Year Entrance Examination

Mathematics Qualifying Sample Examination
August 2019

Attempt ALL QUESTIONS
All questions carry equal marks

Duration of exam: 3 hours.

Total Marks: 150

Minimum Pass Marks: 75

Attachments: NCI Undergraduate Formulae Booklet.

Note: Calculators may be used.

Section: Probability and Statistics

Question 1:

(a) A ball is drawn at random from a box containing 6 red balls, 4 white balls and 5 blue balls. Determine the probability that the ball drawn is

- (i). Red
- (ii). White
- (iii). Blue
- (iv). Not red
- (v). Red or white

(15 marks)

(b) Find the probability of 4 turning up at least once in two tosses of a fair die?

(10 marks)

Question 2:

Over the course of a 20 day period, a student keeps a record of the number of phone calls that she receives per day. The results are presented in the following frequency distribution:

Number of Phone Calls Received	Number of Days
1	1
2	5
3	3
4	7
5	4

Calculate the **standard deviation** of the distribution.

(25 marks)

Section 2: Geometry and Trigonometry

Question 3:

(a) Find all the values of x for which $\cos(4x) = \frac{\sqrt{3}}{2}$, where $0^\circ < x \leq 360^\circ$.

(10 marks)

(b) Find the equation of the perpendicular bisector of the line segment [AB], where A is the point $(-14, 10)$ and B is the point $(26, -22)$.

(15 marks)

Section 4: Number Systems

Question 4:

(a) $(4 + 3i)$ is one root of the equation $az^2 + bz + c = 0$ where $a, b, c \in \mathbb{R}$, and $i^2 = -1$. Write the other root.

(10 marks)

(b) Express $z = (3 + 2i)(2 + 2i)$ in polar form and calculate z^4 . Express the results both in polar and rectangular forms.

(15 marks)

Section 5: Algebra

Question 5:

(a) Solve the simultaneous equations:

$$x + y + z = 16$$

$$\frac{5}{2}x + y + 10z = 40$$

$$2x + \frac{1}{2}y + 4z = 21$$

to find the values of x , y and z .

(15 marks)

(b) Given the equation $x^2 + (k - 2)x + (k - 3) = 0$

(i). Show that the roots are real for all values of $k \in \mathbb{R}$.

(ii). Find the roots of the equation in terms of k .

(10 marks)

Section: Functions

Question 6:

(a) A is the closed interval $[0, 5]$. That is, $A = \{x \mid 0 \leq x \leq 5, x \in \mathbb{R}\}$. The function f is defined on by

$$f: A \rightarrow \mathbb{R} \text{ with } x \mapsto x^3 - 5x^2 + 3x + 5.$$

(i). Find the maximum and minimum values of x .

(ii). State whether f is *injective*. Give a reason for your answer.

(15 marks)

(b) The equation of a circle is $x^2 + y^2 = 20$. Find $\frac{dy}{dx}$ and hence find the slope of the tangent to the circle at the point $(2, 4)$.

(10 marks)