THE MAKING OF ZERO CURVES

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Introduction

Motivation

- Reliable estimate of *risk-free* zero rates for our equity-based credit-risk model [PDSM: Perpetual-Debt Structural Model].

Puzzle

- Negative interest rates. "Pure" interest rates on money loans *cannot* be negative. The role of the "storage cost of money".

Goal

- To adopt a methodology that is *transparent*, assures the "perfect fit" of traded contacts and is a "transaction-based industry standard".

Perpetual-Debt Structural Model

- In our Perpetual-Debt Structural Model (PDSM), we have to value a perpetual American put option to default that depends on an endogenous default trigger (chosen by stockholders to maximize the value of equity) and a perpetual "first-touch" digital option.

- The estimate of a reliable *risk-free zero curve* is critical because, as first noted by Cox and Ross [1976], the value of any *contingent claim* can be derived in a risk-neutral world, where the expected rate of return on assets is equal to the *risk-free* interest rate.

Negative Interest Rates

- An interest rate is the percentage of principal charged by the lender for the use of one of its *assets* (gold, securities, flats, etc.), *not only money*.

- The interest rate *on money* depends, in particular, on the creditworthiness of the borrower and the storage cost of money.

- Why should one lend money at a negative rate, so receiving back - in the future - an amount lower than the amount currently owned? "Pure" interest rates on money *cannot* be negative.

- However, when the interest rate on money is ultra low and the lender's storage cost of money is higher then that of the borrower, the lender may find it convenient to enter into a contract with a negative rate in order to *transfer* the storage costs to the borrower.

Storage Cost of Money

- The storage cost of money can be seen as the cost of an insurance policy paid to the borrower (e.g. a bank) for taking care of the lender's (e.g. a depositor's) money.

- Bank safe deposit boxes are a popular instrument for avoiding negative interest rates.

- Example: If you hold 50,000 Swiss francs in a bank safe deposit box with a 100-franc annual rental fee, you are effectively paying negative interest at the rate of 0.2% per annum. In this case, holding money in a safe deposit box would only be more profitable than investing it in a bank account if the negative interest charged by the bank were higher than 0.2%. [Source: Moneyland Magazine]

The Market for Interest-Rate Derivatives

- The market for interest-rate derivatives is huge. At the end of June 2020, the notional capital of interest-rate derivatives was equal to $495 trillion {Bank for International Settlements (2020)}.

- The bulk of this amount is represented by interest rate swaps ($364 trillion). Therefore, these contracts should play a critical role in the making of zero curves.

ISDA and IHS Markit

- The ISDA CDS Standard Model requires the determination of *risk-free zero curves* for discounting the payments of credit default swaps, with tenors (3M, 6M, 1Y, …, 30Y) that are currency specific.
- This is the "transaction-based industry standard" that we looked for.
- IHS Markit (formerly Markit) the administrator of this open source project, publishes deposit and swap rates, on every weekday, for ten currencies (USD, CAD, EUR, GBP, JPY, CHF, AUD, NZD, HKD, and SGD).
- The website is https://www.markit.com/news/.
- For example, the EUR zip file published on 14 May 2021 contains the interest rates shown in Table 6.

IHS Markit: Interest rates

Table 6	IHS Markit: Interest rates (14 May 2021)			
Snap time	Spot date	Tenor	Maturity date	Par Rate
14/05/2021 14:00	19/05/2021	1M	18/06/2021	-0.005671
14/05/2021 14:00	19/05/2021	3M	18/08/2021	-0.005691
14/05/2021 14:00	19/05/2021	6M	18/11/2021	-0.005717
14/05/2021 14:00	19/05/2021	1Y	18/05/2022	-0.005720
14/05/2021 14:00	19/05/2021	2Y	18/05/2023	-0.005570
14/05/2021 14:00	19/05/2021	3Y	20/05/2024	-0.005092
14/05/2021 14:00	19/05/2021	4Y	19/05/2025	-0.004421
14/05/2021 14:00	19/05/2021	5Y	18/05/2026	-0.003691
14/05/2021 14:00	19/05/2021	6Y	18/05/2027	-0.002961
14/05/2021 14:00	19/05/2021	7Y	18/05/2028	-0.002202
14/05/2021 14:00	19/05/2021	8Y	18/05/2029	-0.001452
14/05/2021 14:00	19/05/2021	9Y	20/05/2030	-0.000751
14/05/2021 14:00	19/05/2021	10Y	19/05/2031	-0.000071
14/05/2021 14:00	19/05/2021	12Y	18/05/2033	0.001149
14/05/2021 14:00	19/05/2021	15Y	19/05/2036	0.002569
14/05/2021 14:00	19/05/2021	20Y	20/05/2041	0.003818
14/05/2021 14:00	19/05/2021	30Y	18/05/2051	0.004077

Bootstrap Method

- The file reported in Table 6 contains 17 interest rates: 4 Euribor rates (1M, 3M, 6M, 12M) and 13 EUR swap rates (2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y, 12Y, 15Y, 20Y, 30Y).

- These rates can be used to build the EUR zero curve.

- In a series of articles published on the web, Mark Rotchell (rotchvba.wordpress.com) has explored the ISDA CDS Standard Model.

- Markit uses the *bootstrap method* to convert the swap rates into spot prices.

- In particular, we highlighted the assumptions made by Markit about the forward rates for intermediate maturities (Figure 3).

Forward Rates

Figure 3   Forward rates.
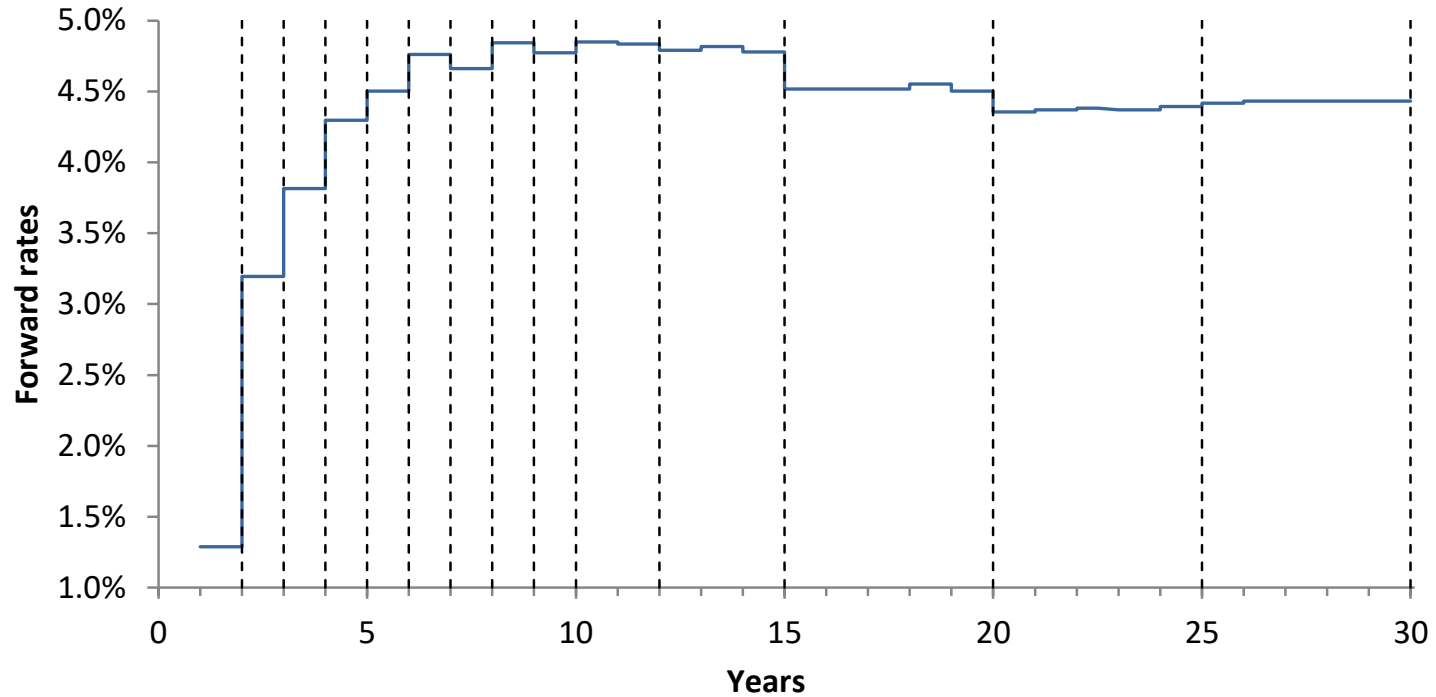
The Market Standard

- The bootstrap method followed by ISDA/Markit is the "market standard" to calculate the fair value of interest rate swaps.

- The method describes each zero curve by a high number of *nodes* or *vertices*, 62 in the EUR case: 4 maturities for deposits up to 1 year and 58 (= 29 × 2) for swaps up to 30 years.

- In other terms, 62 discount factors have been used to perfectly fit the value of 17 contracts (4 deposits plus 13 swaps).

Zero Curves

- A "movie" showing the daily changes of the EUR zero curve from 29 December 2006 to 14 May 2021 is available here:

  http://docenti.luiss.it/barone/files/2021/05/EUR_Zero_Rates_Movie.xlsm.

- Historically, the short end of the zero curve has not been smooth, as in Figure 4 dated 14 May 2020.

- In our time series (2,398 observations) such weird shapes, with a short-end hump (1-year zero rate higher than both the 3-month and 2-year rates) happen 65% of the cases.

A Weird Zero Curve

Figure 4   Zero curve (14 May 2020).
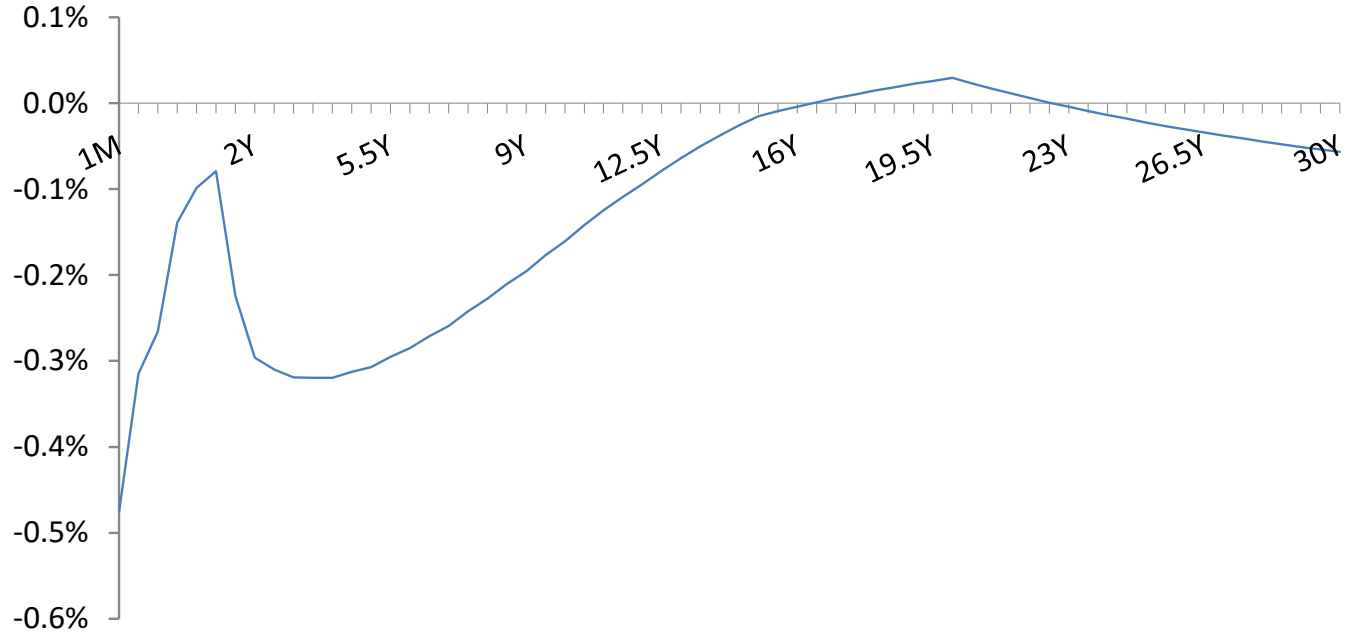
Libor Phase-Out and New Reference Rates

- In July 2017, the Financial Conduct Authority (FCA) announced the discontinuation of Libor. The decision was taken because of the absence of active underlying markets, which requires the use of "expert judgement" by the panel banks to form many of their submissions.

- The Libor panel banks agreed to continue to submit to Libor until end-2021 (subsequently extended to end-June 2023 for US dollar Libor only).

- Following the financial crisis that started in 2007, many dealers switched to using over-night indexed swap (OIS) rates as risk-free rates.

- Libor will be replaced by overnight reference rates such as Ester (*euro short-term rate*) in the Eurozone, Sofr (*secured overnight financing rate*) in the United States, Sonia (*sterling overnight index average*) in the U.K., Saron (*Swiss average rate overnight*) in Switzerland, Tonar (*Tokyo overnight average rate*) in Japan.

Risk-Free Rates (RFRs)

- The use of risk-free rates (RFRs) will become the market convention.

- As a consequence, IHS Markit replaced "old" interest rates with "new" ones.

- Differently from the past, when they were based on Deposit and Swap Rates, the new interest rate curves are now based on "RFR OIS (Swap) rates", where RFR and OIS are the acronyms of Risk Free Rates and Overnight Indexed Swaps, respectively.

- In the case of quotes published by IHS Markit, the frequency of both floating and fixed payments of OISs with life greater than one year is annual.

- As regards the interpolation method, no change has been made with respect to the previous approach.

New and Old Risk-Free Zero Curves: A Comparison

- Starting from July 9th, 2021, IHS Markit is publishing the EUR-denominated interest rate curves based on both the old and new methodology.

- This makes it possible to compare the new and old risk-free zero curves.

- In the period from July 9th to August 20th, 2021, the average difference between the zero rates calculated according to the old methodology and the new one ranges from 0.90 basis points at the 1 month maturity to 16.37 basis points at the 12-year maturity (Table 9).

| Table 9 Average difference between "old" and "new" risk-free zero rates (basis points).																
Maturity	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	30Y
Delta	0.90	2.36	4.93	7.91	8.45	10.42	12.31	13.76	14.70	15.32	15.80	16.16	16.35	16.37	15.87	14.89	13.28

Source: elaborations of IHS Markit data (9 July 2021 to 20 August 2021).

New and Old Risk-Free Zero Curves: A Comparison

- The new EUR-denominated zero curve on August 20th, 2021, is shown in Figure 7.

- In the short end there is no evidence of the "asperities" that have been often observed in the past when the zero curve was based also on Libor rates.

Figure 7   Average difference between "old" and "new" risk-free zero rates.
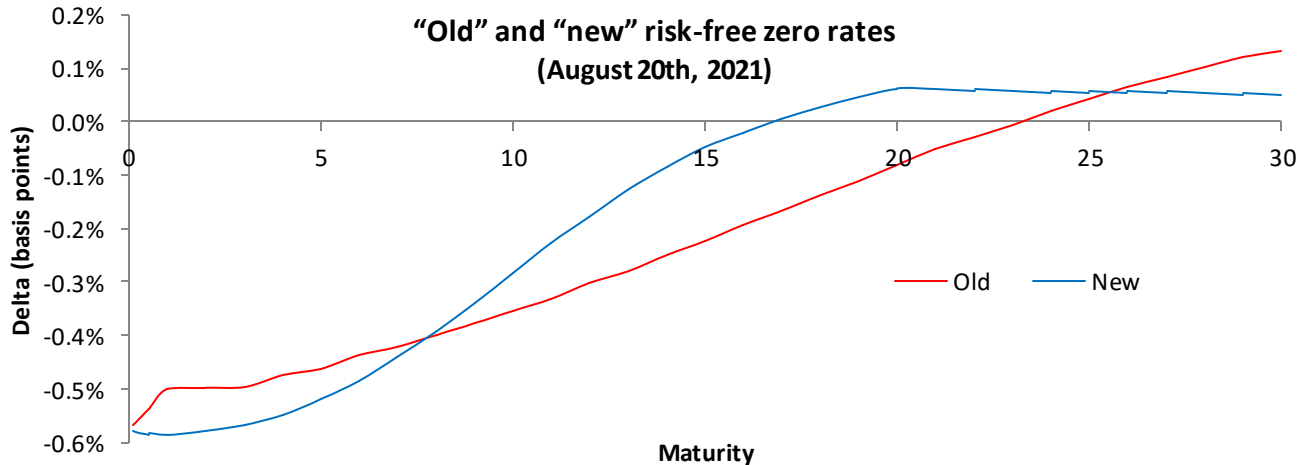
Conclusions

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