A game-theoretical model for green urban logistics coordination: exact and data-driven methods

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Motivation

URBANE: Upscaling Innovative Green Urban Logistics Solutions Through Multi-Actor Collaboration and PI-Inspired Last Mile Deliveries.https://www.urbane-horizoneurope.eu/

- A 3.5-year Horizon Europe project (2022-2026) on novel, **sustainable**, safe, resilient and **effective last-mile delivery solutions**, combining green automated vehicles and shared space utilisation models.
- **Living Labs:**
  Helsinki (FI), Bologna (IT), Valladolid (ES), Thessaloniki (GR)
- **Twinning Living Labs:**
  Barcelona (ES), Karlsruhe (DE)
- **Follower Cities:**
  Aarhus (DK), Antwerp (BE), La Rochelle (FR), Mechelen (BE), Prague (CZ), Ravenna (IT)

Motivation - Challenges of Bologna LL (1)

- Coordinate the activities of the logistics operators
- Reduction of the distance travelled by the delivery vehicles and consequently reduction of the negative externalities produced by them (road congestion, noise, CO_2 and pollutant emissions, etc.)

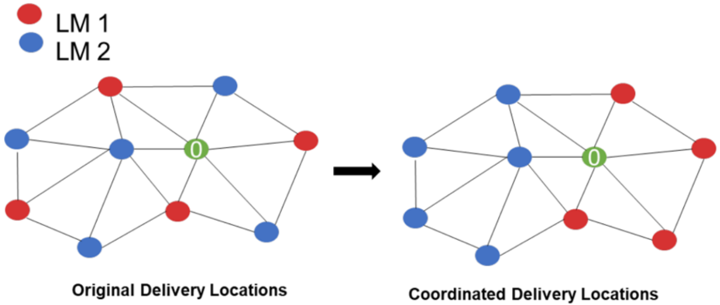


Figure: Solution Example (Last Mile only)

Motivation - Challenges of Bologna LL (2)

Figure: Bologna LL Limited Traffic/Emission Zone (LTZ/LEZ)

Bilevel Optimization for Greener Last Mile Logistics (1)

The goal is to consolidate parcel deliveries outside the LTZ/LEZ, and reduce the overlap of the operating areas of the Delivery Service Providers (DSPs) in the LEZ

Two-echelon (2E) multi-commodity routing model:

- First echelon: First Mile (FM) DSPs which deliver parcels to satellite hubs at the edges of the LEZ
- Second echelon: Last Mile (LM) DSPs must pickup parcels at satellite hubs and deliver them to customers

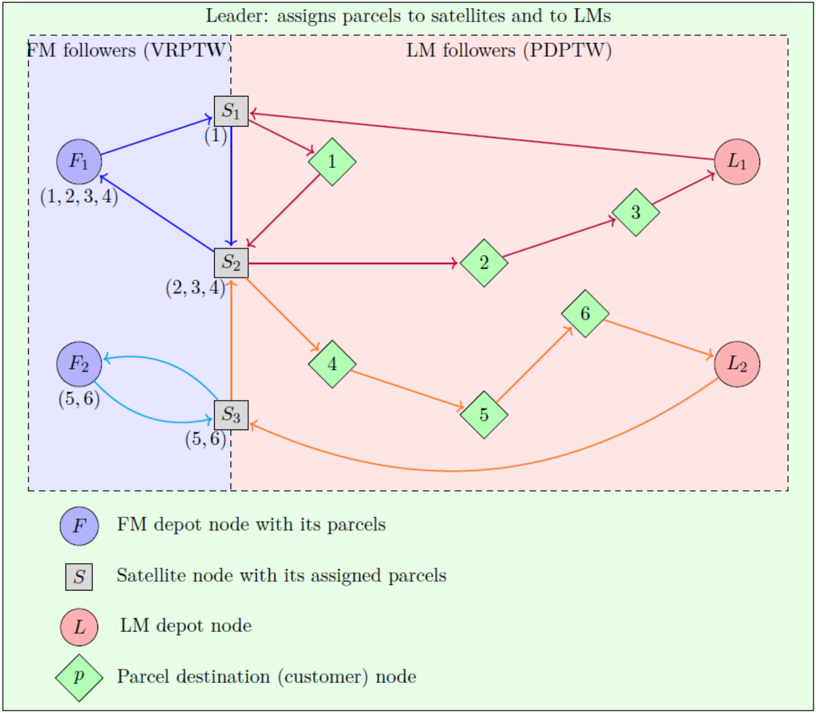
Several non-cooperative agents need to be coordinated $\implies$ Bilevel Optimization (BO) [Dempe, 2002]

Bilevel Optimization for Greener Last Mile Logistics (2)

BO approach:
Stackelberg game where the leader represents a regulator whose goal is to minimize emissions in the LEZ

- The **leader** assigns parcels to satellite hubs and to LM DSPs. The followers seek to minimize their own routing costs and maximize client satisfaction
- Each **FM follower** solve a Vehicle Routing Problem with Time Windows (VRPTW)
- Each **LM follower** solves a pickup-and-delivery VRPTW (PDPTW)

$\Longrightarrow$ **Single leader, multi-follower bilevel optimization formulation** for 2E multi-commodity routing problem to facilitate the city's goals of reducing emissions in its LEZ



Leader: assigns parcels to satellites and to LMs
FM followers (VRPTW)
LM followers (PDPTW)
S_1
(1)
F_1
(1, 2, 3, 4)
1
L_1
3
S_2
(2, 3, 4)
2
6
F_2
(5, 6)
4
L_2
S_3
(5, 6)
5
F
FM depot node with its parcels
S
Satellite node with its assigned parcels
L
LM depot node
p
Parcel destination (customer) node

Leader Problem

Minimize emissions in both echelons:

$$
\begin{aligned}
\min_{\boldsymbol{\lambda},\boldsymbol{y}} \quad & \sum_{k\in\mathcal{K}_1}\sum_{(i,j)\in\mathcal{A}_1} E_{ij}^k x_{ij,f}^k + \sum_{k\in\mathcal{K}_2}\sum_{(i,j)\in\mathcal{A}_2} E_{ij}^k x_{ij,d}^k \\
\text{s.t.} \quad & \sum_{f\in\mathcal{F}}\sum_{p\in\mathcal{P}_f} \lambda_p^s \le C_s && \forall s\in\mathcal{S} \\
& \sum_{s\in\mathcal{S}} \lambda_p^s = 1 && \forall p\in\mathcal{P} \\
& \sum_{d\in\mathcal{D}} y_p^d = 1 && \forall p\in\mathcal{P} \\
& G(\boldsymbol{\lambda},\boldsymbol{y}) \ge 0 \\
& \lambda_p^s \in \{0,1\} && \forall p\in\mathcal{P}, s\in\mathcal{S} \\
& y_p^d \in \{0,1\} && \forall p\in\mathcal{P}, d\in\mathcal{D} \\
& (\boldsymbol{x}_f,\boldsymbol{t}_f) \in \mathsf{VRPTW}(\boldsymbol{\lambda}_f) && \forall f\in\mathcal{F} \\
& (\boldsymbol{x}_d,\boldsymbol{t}_d) \in \mathsf{PDPTW}(\boldsymbol{\lambda},\boldsymbol{y}_d) && \forall d\in\mathcal{D}
\end{aligned}
$$

First Mile Follower Problem

Each FM follower $f \in \mathcal{F}$ minimizes the total route cost associated with delivering its set of parcels $\mathcal{P}_f$

$$
\begin{aligned}
\text{VRPTW}(\boldsymbol{\lambda}_f) : \min_{\boldsymbol{x}_f, \boldsymbol{t}_f} \quad & \sum_{k \in \mathcal{K}_f} \sum_{(i,j) \in \mathcal{A}_1} c_{ij} x_{ij,f}^k \\
\text{s.t.} \quad & \sum_{i \in \mathcal{V}_1} \sum_{k \in \mathcal{K}_f} x_{is,f}^k \geq \lambda_p^s && \forall p \in \mathcal{P}_f, s \in \mathcal{S} \\
& \sum_{s \in \mathcal{S}} x_{D_f s,f}^k \leq 1 && \forall k \in \mathcal{K}_f \\
& \sum_{i \in \mathcal{V}_1} x_{ij,f}^k - \sum_{i \in \mathcal{V}_1} x_{ji,f}^k = 0 && \forall k \in \mathcal{K}_f, j \in \mathcal{V}_1 \\
& t_{i,f}^k + T_{ij} \leq t_{j,f}^k + (1 - x_{ij,f}^k) M_{ij} && \forall k \in \mathcal{K}_f, (i,j) \in \mathcal{A}_1 \\
& e_i \sum_{j \in \mathcal{V}_1} x_{ij,f}^k \leq t_{i,f}^k \leq l_i \sum_{j \in \mathcal{V}_1} x_{ij,f}^k && \forall k \in \mathcal{K}_f, i \in \mathcal{V}_1 \\
& x_{ij,f}^k \in \{0,1\} && \forall k \in \mathcal{K}_f, (i,j) \in \mathcal{A}_1 \\
& t_{i,f}^k \geq 0 && \forall k \in \mathcal{K}_f, i \in \mathcal{V}_1
\end{aligned}
$$

Last Mile Follower Problem (1)

Each LM follower $d \in \mathcal{D}$ minimizes its total routing cost, as well as time window violations

$$\begin{aligned}
\text{PDPTW}(\boldsymbol{\lambda}, \boldsymbol{y}_d) : \min_{\boldsymbol{x}_d, \boldsymbol{t}_d} \quad & \sum_{k \in \mathcal{K}_d} \sum_{(i,j) \in \mathcal{A}_2} c_{ij} x_{ij,d}^k \\
& + \pi^L \sum_{k \in \mathcal{K}_d} \sum_{i \in \mathcal{V}_2} \max\{t_{i,d}^k - l_i \sum_{j \in \mathcal{V}_2} x_{ij,d}^k, 0\} \\
& + \pi^E \sum_{k \in \mathcal{K}_d} \sum_{i \in \mathcal{V}_2} \max\{e_i \sum_{j \in \mathcal{V}_2} x_{ij,d}^k - t_{i,d}^k, 0\}
\end{aligned}$$

Last Mile Follower Problem (2)

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{j \in \mathcal{V}_2} x^k_{j\delta(p),d} = y^d_p && \forall p \in \mathcal{P}, \forall k \in \mathcal{K}_d \\
& \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{V}_2} \lambda^s_p x^k_{sj,d} \geq y^d_p && \forall p \in \mathcal{P}, \forall k \in \mathcal{K}_d \\
& \sum_{s \in \mathcal{S}} x^k_{D_d s,d} \leq 1 && \forall k \in \mathcal{K}_d \\
& \sum_{j \in \mathcal{V}_2} x^k_{sj,d} \leq 1 && \forall k \in \mathcal{K}_d, s \in \mathcal{S} \\
& \sum_{i \in \mathcal{V}_2} x^k_{ij,d} - \sum_{i \in \mathcal{V}_2} x^k_{ji,d} = 0 && \forall k \in \mathcal{K}_d, j \in \mathcal{V}_2 \\
& \sum_{s \in \mathcal{S}} \lambda^s_p \left(t^k_{s,d} + T_{s\delta(p)} \right) \leq t^k_{\delta(p),d} + (1 - y^d_p) N^k_p && \forall p \in \mathcal{P}, k \in \mathcal{K}_d \\
& t^k_{i,d} + x^k_{ij,d} T_{ij} \leq t^k_{j,d} + (1 - x^k_{ij,d}) M_{ij} && \forall k \in \mathcal{K}_d, (i,j) \in \mathcal{A}_2 \\
& x^k_{ij,d} \in \{0,1\} && \forall (i,j) \in \mathcal{A}_2, k \in \mathcal{K}_d \\
& t^k_{i,d} \geq 0 && \forall i \in \mathcal{V}_2, k \in \mathcal{K}_d
\end{aligned}
$$

Solution Approach 1 — Cutting Plane Algorithm

Initialization: relax followers' best-response constraints in Leader problem $\implies$ High-Point Relaxation (HPR) and reformulate HPR as MILP ($\max$ and bilinear term linearizations)

At iteration n:

1. Solve HPR to obtain lower bound (LB), candidate $(\boldsymbol{\lambda}^n, \boldsymbol{y}^n)$ solution and estimates of follower's objective function values (OFV): $\hat{V}_f^n$, $\hat{V}_d^n$
2. Solve $(\boldsymbol{\lambda}^n, \boldsymbol{y}^n)$-parameterized follower problems to obtain upper bound (UB) and best-responses OFVs: V_f^n, V_d^n
3. **Optimality cuts generation**:
   - if $\hat{V}_f^n - V_f^n > \epsilon$ generate FM optimality cut for FM f
   - if $\hat{V}_d^n - V_d^n > \epsilon$ generate LM optimality cut for LM d
4. Add **interdiction cut**

Exact cutting plane algorithm that terminates if LB and UB converge, **or** if no optimality cuts are added

Solution Approach 2 — Constraint Learning (CL)

In BO, we work for the leader: follower problems are the leader's representations of FM and LM DSPs' operations

Motivation for a CL heuristic approach:

- Leader decisions are **parcel hub and LM assignments**, precise DSP routes are not the target output
- Data-driven **prediction** of FM and LM best responses may be sufficient to find efficient leader decisions
- **Training** predictive models for learning FM and LM best responses consists of solving multiple parameterized VRPTWs and PDPTWs for which advanced algorithms are available in the literature

CL-Based Leader Problem

$$
\begin{aligned}
\min_{\boldsymbol{\lambda},\boldsymbol{y},\boldsymbol{\gamma}} \quad & \sum_{f\in\mathcal{F}} E_f + \sum_{d\in\mathcal{D}} E_d + \rho_S \sum_{s\in\mathcal{S}} \gamma_s + \rho_D \sum_{d\in\mathcal{D}} \gamma_d \\
\text{s.t.} \quad & \sum_{f\in\mathcal{F}} \sum_{p\in\mathcal{P}_f} \lambda_p^s \le C_s & \forall s \in \mathcal{S} \\
& \sum_{s\in\mathcal{S}} \lambda_p^s = 1 & \forall p \in \mathcal{P} \\
& \sum_{d\in\mathcal{D}} y_p^d = 1 & \forall p \in \mathcal{P} \\
& E_f = \mathsf{Pred}(\boldsymbol{\lambda}_f) & \forall f \in \mathcal{F} \\
& E_d = \mathsf{Pred}(\boldsymbol{\lambda}, \boldsymbol{y}_d) & \forall d \in \mathcal{D} \\
& \gamma_s \ge \lambda_p^s & \forall p \in \mathcal{P}, s \in \mathcal{S} \\
& \gamma_d \ge y_p^d & \forall p \in \mathcal{P}, d \in \mathcal{D} \\
& \lambda_p^s \in \{0,1\} & \forall p \in \mathcal{P}, s \in \mathcal{S} \\
& y_p^d \in \{0,1\} & \forall p \in \mathcal{P}, d \in \mathcal{D}
\end{aligned}
$$

$\mathsf{Pred}(\cdot)$ constraints can be formulated using MILP [Maragno et al., 2023]

ρ_S and ρ_D are design penalties

Numerical Experiments — Setup

All models are implemented in Python:

- Exact approach implemented using CPLEX 22.1
- Data-driven approach implemented using Gurobi and `gurobi-machinelearning 1.4.0`

We compared 3 approaches:

- Uncoordinated deliveries (heuristic $(\boldsymbol{\lambda}, \boldsymbol{y})$ assignment)
- Fully Coordinated deliveries (Bilevel: Exact)
- Fully Coordinated deliveries (Bilevel: Data-Driven)

Numerical Results — Unassigned Deliveries

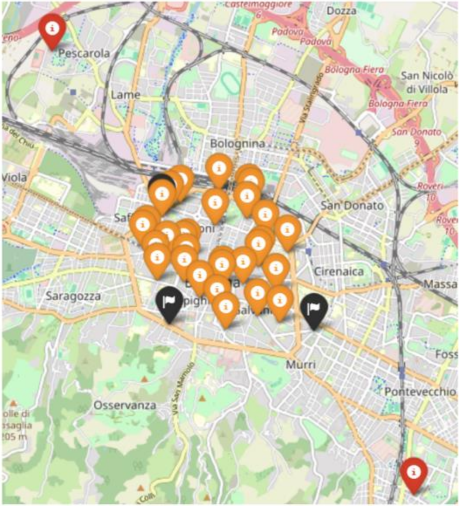


Figure: Problem instance with $|\mathcal{D}| = 2$, $|\mathcal{S}| = 3$ & $|\mathcal{P}| = 30$

Numerical Results — Assigned and Coordinated Deliveries

Figure: Coordinated approach leads to consolidation of delivery locations (berlin_2_2_3_30) (only last mile solution shown)

Numerical Results — Optimization Methods

Table: Performance of different modelling approaches on the Madrid instances of the bilevel 2E problem. Iter. denotes the number of iterations; Time is the runtime in seconds; Obj. is the objective value of the best feasible solution found; Gap is the optimality gap upon termination in percentage.

Method	Exact				Data-Driven		Uncoordinated	
Instance	Obj.	Time	Iter.	Gap	Obj.	Time	Obj.	Time
madrid_2_2_2_10	**405.97**	50.23	3	0.00	**405.97**	0.48	656.76	0.40
madrid_2_2_2_30	828.88	4225.72	4	37.06	**824.11**	306.49	1177.42	303.13
madrid_2_2_2_50	1309.35	3916.70	4	51.17	**1283.01**	348.77	1684.25	304.78
madrid_2_2_3_10	**431.24**	705.12	5	0.00	438.29	0.64	628.00	0.59
madrid_2_2_3_30	**794.48**	4279.60	4	40.36	842.87	344.56	1169.94	302.16
madrid_2_2_3_50	1414.53	3936.97	4	58.09	**1352.54**	507.74	1656.34	309.48
madrid_2_3_2_10	**405.97**	144.48	4	0.00	460.10	1.34	760.02	1.25
madrid_2_3_2_30	**852.85**	3617.98	4	40.74	853.64	312.00	1360.28	604.82
madrid_2_3_2_50	1398.35	4283.50	4	56.05	**1334.38**	391.53	1514.09	613.17
madrid_2_3_3_10	**431.24**	3796.13	9	0.00	438.29	1.30	968.20	1.56
madrid_2_3_3_30	856.77	2709.28	4	46.41	**844.46**	472.39	1312.28	605.92
madrid_2_3_3_50	1329.87	4528.09	4	56.04	**1224.56**	550.30	2030.28	643.99

Numerical Results — Total Emissions

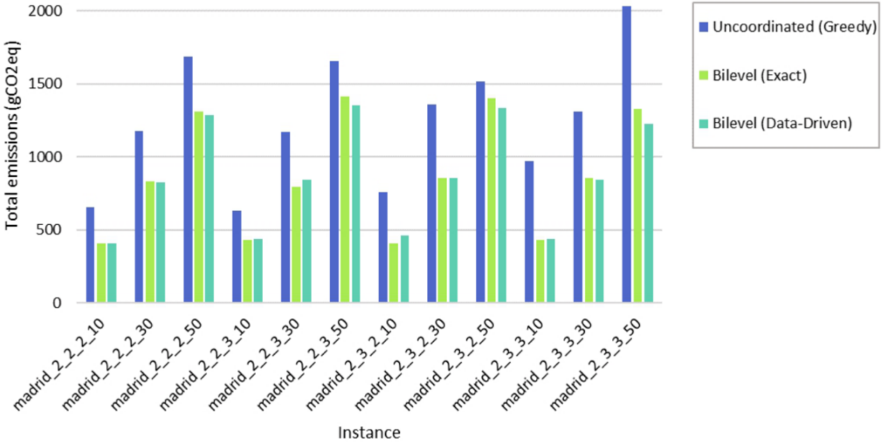


Figure: Total emissions (gCO2eq) across all Madrid instances.

Summary and Conclusions

- New collaborative delivery model for greener urban logistics.
- Bilevel optimization approach where followers represent delivery service providers.
- Exact and data-driven solution methods are developed and implemented.
- Coordinated model is shown to strike a balance between carbon emissions reduction and delivery service providers operating costs.
- Constraint learning-based approach provides a tractable solution method for optimizing city-scale operations.
- Reduced daily operational emissions by 15%

References

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